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MULTIPLE MODES OF VORTEX-INDUCED VIBRATION OF A SPHERE

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In this work, we discover the existence of multiple modes of vortex-induced vibration of a tethered sphere in a free stream. In addition to the first two modes, defined as Modes I and II, and found originally by Govardhan & Williamson (1997), we find the existence of an unexpected Mode III at much higher normalized velocities (U^*). This third mode, involving large-amplitude and remarkably periodic vibrations, was discovered by changing our focus from "light", or buoyant, tethered spheres in a water facility (where relative density, $m^* < 1$), to "heavy" spheres in wind tunnel facilities (where $m^* \gg 1$). In this manner, we are able to achieve a very wide range of normalized velocities, $U^* = 0 - 300$, and investigate a wide range of masses, $m^* = 0{\cdot}1 - 1000$. The first two modes might be identified as analogies to the 2S and 2P modes for an excited cylinder (Williamson & Roshko 1988), and can be associated with a lock-in of the vortex formation frequency with the natural frequency. These modes of sphere dynamics occur within the velocity regime $U^* \sim 5 - 10$. However, our Mode III occurs over a broad range of high velocity ($U^* \sim 20 - 40$), where the body dynamics cannot be synchronised with the principal vortex formation frequency. At extremely high velocities ($U^* > 100$), we find yet another mode of vibration that persists to at least $U^* > 300$, which we define as Mode IV, but in this case the unsteady oscillations are characterized by intermittent bursts of vibration. Regarding the periodic Mode III, it cannot be explained by classical "lock-in" of the principal vortex shedding and body motion, and one is left with a tantalizing question: *What causes this unexpected periodic high-speed mode of vortex-induced vibration?*

1. INTRODUCTION

The case of a tethered sphere vibrating in a fluid flow is perhaps one of the most basic fluid-structure interactions that one can imagine. By a wide variation of the mass of the sphere, one can consider the case of an underwater tethered buoyant body, or indeed a "heavy" sphere in air flow, acting as a pendulum, as examples of essentially the same general problem. It is quite surprising that, despite the fact that tethered bodies are quite ubiquitous in engineering, very few investigations have shown whether a tethered sphere will oscillate in a steady fluid flow or current. It was demonstrated by Williamson & Govardhan (1997) and Govardhan & Williamson (1997), that such a structure will indeed vibrate vigorously at large amplitude, and these oscillations have a direct impact on the tether angle and drag coefficient. Gross errors in predictions of mean response of a tethered structure will ensue unless one takes account of their tendency to vibrate. Other studies in the literature are concerned with the action of surface waves on tethered buoyant structures, and they employ empirically-determined drag and inertia coefficients to predict sphere dynamics (Harleman & Shapiro 1961; Shi-Igai & Kono 1969; Ogihara 1980).

In this work, we define a sphere as either "light" or "heavy", depending on the value of the relative density or mass ratio, m^* (where m^* = mass of sphere/mass of displaced fluid):

"*Light*" *sphere*: $m^* < 1$;

"*Heavy*" *sphere*: $m^* > 1$.

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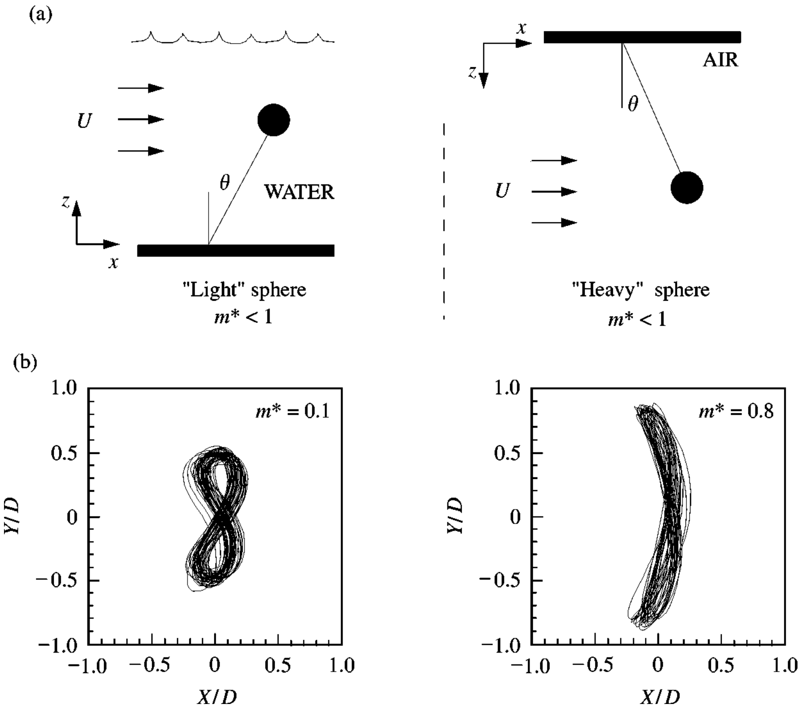

Figure 1. (a) Schematic of the experimental arrangement for the "light" and "heavy" tethered sphere. (b) Typical trajectories of sphere motion for "light" spheres.

Most of the studies in Govardhan & Williamson (1997) involved "light" spheres, where the tethered bodies in our Water Channel facility were buoyant, and typical trajectories were found to be of figure-of-eight or crescent topologies, as shown in Figure 1. The oscillation amplitudes transverse to the fluid flow (y-direction) are always found to be much larger than streamwise motions (x-direction), especially as the spheres become "heavy". It was shown in Williamson & Govardhan (1997) that the normalized amplitude ($A^* = A/D =$ amplitude/diameter) can be suitably collapsed using the normalized velocity $U^* = U/f_N D$ (where U is the free-stream velocity, f_N the natural frequency in the fluid, and D the diameter), as could be expected on dimensional grounds. In the case of the vortex-induced vibration of a cylinder, such response plots show a resonance when the vortex shedding frequency f_V is close to the natural frequency of the structure f_N, which corresponds to a velocity $U^* \sim 1/S \approx 5$, where S is the Strouhal number. The response of the sphere, in Figure 2, shows just such a resonance at $U^* \approx 6$, which we define as the Mode I response, and this corresponds to the vortex formation frequency lying close to the (calculated) natural frequency of the tethered body.

At higher velocities ($U^* > 8$), a Mode II periodic vibration appears, with large amplitudes close to one diameter, and in the case of the low mass, $m^* = 0{\cdot}8$, the extent of the synchronization regime (the range in U^* over which large vibrations are observed) seems to persist to the limits of our facility, i.e. to at least $U^* > 15$. It is known that increasing the mass of a vibrating structure will decrease the sychronization regime; in the case of the vibrating cylinder, predictions of this effect can be made (Govardhan & Williamson 2000). In the case of sphere dynamics, by increasing the mass from $m^* = 0{\cdot}8$ to $2{\cdot}8$, the end of the synchronization regime reduces to $U^* = 11$, as shown in Figure 2, and thus can be reached

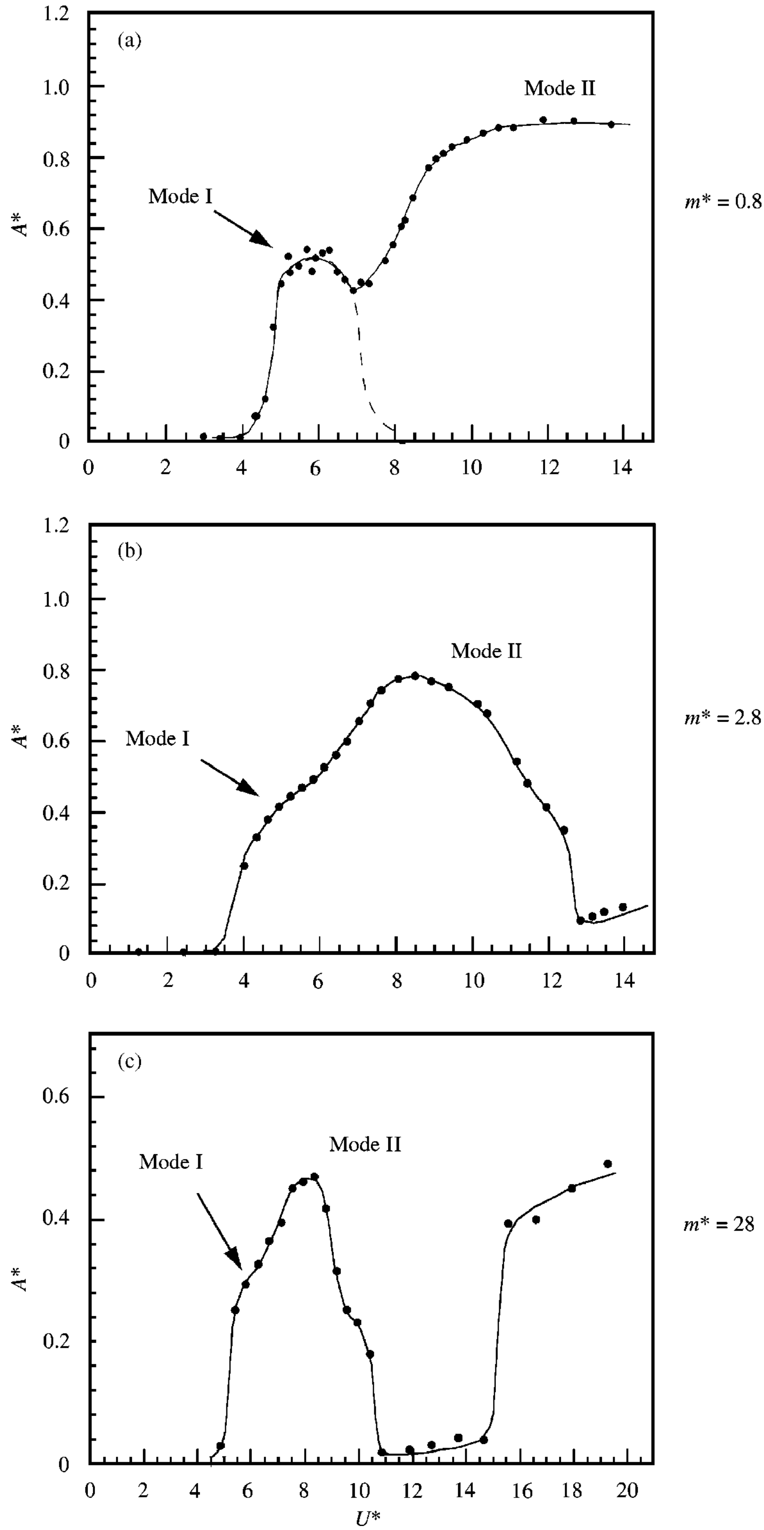

Figure 2. Amplitude response (A^*) versus normalized flow velocity (U^*), showing Modes I and II sphere oscillations: (a) $m^* = 0{\cdot}8$, (b) $m^* = 2{\cdot}8$, (c) $m^* = 28$.

within the flow speed limits of the water facility. It was naturally felt that any further increase in fluid velocity, beyond the limit of the Mode II regime (in the latter case, beyond $U^* = 11$), would yield negligible amplitudes, and exhibit no further modes of response. However, one particular "strange" result from the Water Channel for $m^* = 28$ showed an upsurge in the amplitude close to the limits of the facility flow speed, and beyond the Mode II regime, as shown in Figure 2. Our assumption for some months was that this was a problem with possible flow turbulence, as the channel was operating at "flat out" speed. In order to shed light on this problem, we decided to shift our efforts to wind tunnels, and purposely increase the mass ratio, m^*, beyond the maximum value possible in a water facility, and to enable much higher flow speeds to be studied, thereby permitting a much larger range of normalized velocities, U^*.

This paper presents the principal results from the research performed using the wind tunnels. We have been able to use large mass ratios, from $m^* = 80$ up to 940, and we have achieved a very wide range of normalized velocity, $U^* = 0$–300. We shall see that not only was the upsurge in amplitude, mentioned above, a "real" effect, but that it is part of a wide regime of highly periodic large-amplitude vibrations. This is completely unexpected, because, at these high speeds, the vortex formation frequency is far above the vibration frequency, such that several cycles of vortex structures will be formed over a single period of body motion, and therefore the classical "lock-in" cannot occur. At higher velocities, beyond $U^* = 100$, we discover yet another mode of response, but in this case the oscillations are highly unsteady, and they occur in intermittent bursts.

2. EXPERIMENTAL DETAILS

The experiments described in the Introduction, which utilized the Cornell-ONR Water Channel, were conducted as part of Govardhan & Williamson (1997, 2000) and are described in detail therein. The wind tunnel experiments here involve the use of a 12 in × 12 in (test-section) wind tunnel, and an 18 in × 18 in tunnel (1in = 25·4 mm). Typical spheres in this study had diameters of 6·9 and 7·6 cm, and had masses of 16·5 and 259·1 g, giving mass ratios of 80 and 940, and were tethered to the roof of the tunnel using fine polymer wires, of 0·001 in diameter. Displacement was measured using an optical biaxial displacement transducer, which was oriented upwards from beneath the wind tunnel lucite floor. We shall define the normalized amplitude of the transverse (y) oscillations, unless otherwise noted, as $A^* = \sqrt{2}\, y_{\rm rms}/D$, which, for purely sinusoidal oscillations, is simply $A^* = A/D$.

3. DISCOVERY OF MODE "III": A HIGH-VELOCITY RESPONSE MODE

By using the wind tunnels, we are able to explore the sphere dynamics at high normalized velocities, beyond the regimes of Modes I and II, using a sphere of mass, $m^* = 80$. We discover a new and unexpected mode of vibration, which we define as Mode III, and which is shown clearly in Figure 3, extending in a very broad regime of U^* from 20 to 40. This shows immediately that the upturn of data found in the Water Channel at the upper limit of flow speeds is in fact a real effect, and was a rather serendipitous signal of the beginnings of a significant regime of periodic vibrations, which we might otherwise have overlooked. However, suspicion remained that the results, though apparently real, could be related with the proximity of the sphere to the tunnel walls, or with the blockage of the sphere in the test section. For this reason, we relocated the complete experiment from the 12 in × 12 in tunnel to a larger tunnel of cross-section 18 in × 18 in, and found good agreement between the data

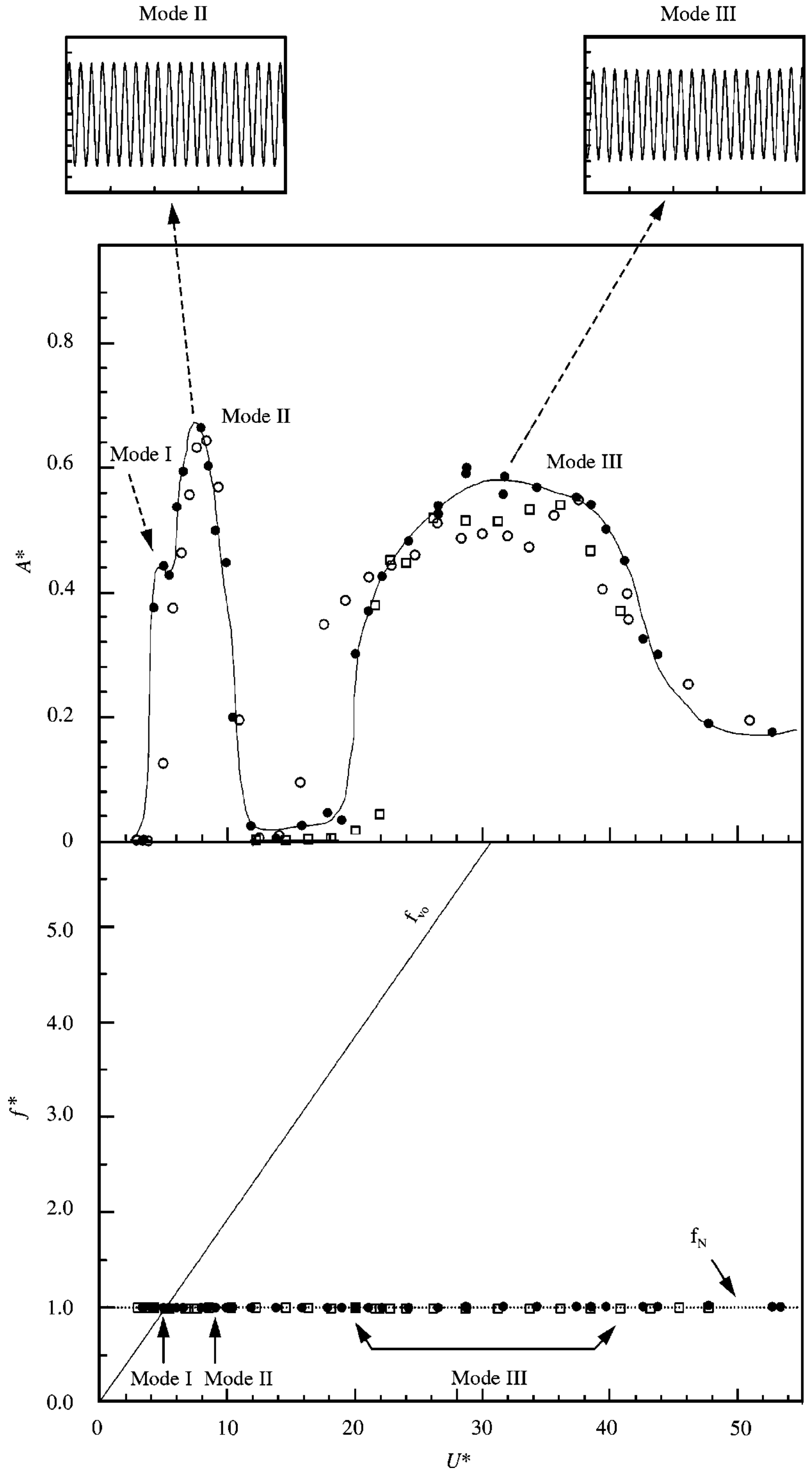

Figure 3. Amplitude (A^*) and frequency (f^*) response over a large range of U^*, showing the very broad regime of periodic Mode III oscillations ($U^* = 20$–40). The oscillation frequency (f) remains close to the natural frequency (f_N) over the observed range of U^*; f_{vo} is the non-oscillating shedding frequency. ●, $m^* = 80$ (12 in × 12 in wind tunnel); ○, $m^* = 80$ (18 in × 18 in wind tunnel); □, $m^* = 940$ (18 in × 18 in wind tunnel).

of the two tunnels. Finally, we studied the dynamics of a very heavy sphere of mass ratio, $m^* = 940$, in the larger tunnel, and found again the Mode III response regime, as shown in Figure 3. These experiments provide good evidence for the existence of this mode of vibration.

The sphere dynamics of Mode III are remarkably periodic, as indicated by the typical time traces of displacement in Figure 3. The oscillation frequency (f) remains very close to the natural frequency (f_N) of the tethered sphere (i.e. $f^* = f/f_N \approx 1$), which is a consequence of the high mass of the spheres in this case. [Very low mass ratios yield oscillation frequencies which are higher than, and depart significantly from, the natural frequency, as shown in Govardhan & Williamson (1997)]. The existence of such a Mode III was completely unforeseen, because such a regime does not exist in the case of the cylinder free vibration, and also because it must reflect the presence of between 3 and 8 wavelengths of vortex formation, for each wavelength of body vibration. Therefore, this mode cannot be explained as a classical "lock-in" of the principal vortex shedding frequency with the body oscillation frequency.

For this high-speed mode of vibration to exist, there must be a net energy transfer from the fluid motions to the body motions, over each cycle of sphere oscillation. If one assumes that the transverse displacement (y) and force (F) are represented by the following functions:

$$y(t) = A\sin(\omega t),$$

$$F(t) = F_o \sin(\omega_o t + \phi)$$

and that the system damping and stiffness are linear, then one can simply show that the net energy transfer over a cycle of body oscillation ($E_{\rm in}$) is given by

$$E_{\rm in} = (F_o A\,\omega)\sin\phi \int_0^T \cos(\omega t)\cos(\omega_o t)\,{\rm d}t.$$

This integral is only nonzero if $\omega = \omega_o$. In other words, as one might expect, there is only energy transfer if there is a periodic component of the fluid force synchronized with the body oscillation frequency. The principal vortex structures are formed at a frequency much higher than the body oscillation frequency, and these cannot be expected to contribute to the body dynamics.

However, there must exist vortex dynamics which are repeatable in each cycle, and which give rise to the fluid forcing component that is synchronized with the body motion.

One should note that it is possible that the vortex shedding is modulated by the low-frequency body motion, such that self-excited motion will ensue. Although the existence of this Mode III is reported in this paper, the vorticity dynamics, which would explain its existence, will be explored in Govardhan & Williamson (2001), using the DPIV technique.

4. MODE "IV": INTERMITTENT BURSTS OF LARGE AMPLITUDE VIBRATION

With further increase of normalized flow speed, beyond the regime for Mode III, one might finally expect negligible response amplitude, and this is the case until about $U^* = 100$. However, beyond this speed, we discovered yet another vibration mode, that grows in amplitude and persists to the limit in flow speed of our wind tunnel (in excess of $U^* = 300$), as shown in Figure 4. (We must expect that the amplitude of this mode will ultimately saturate at sufficiently high velocity.) In this case, the sphere dynamics are not close to periodic, and are characterized by intermittent bursts of large-amplitude vibration, as may be seen in the typical displacement time traces in Figure 4, at $U^* = 120$ and 220. These

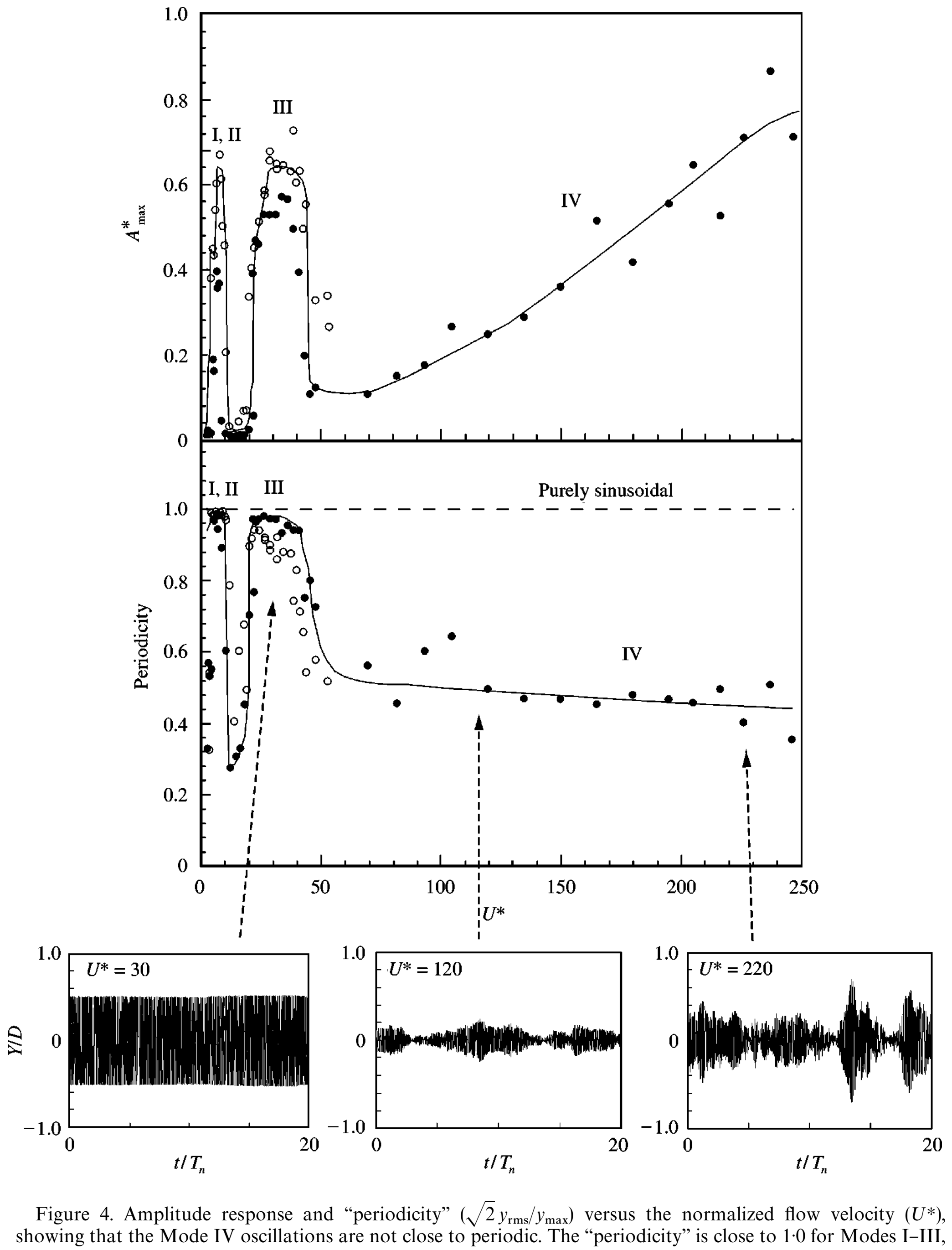

Figure 4. Amplitude response and "periodicity" ($\sqrt{2}\,y_{rms}/y_{max}$) versus the normalized flow velocity (U^*), showing that the Mode IV oscillations are not close to periodic. The "periodicity" is close to 1·0 for Modes I–III, indicating that these modes are quite sinusoidal.

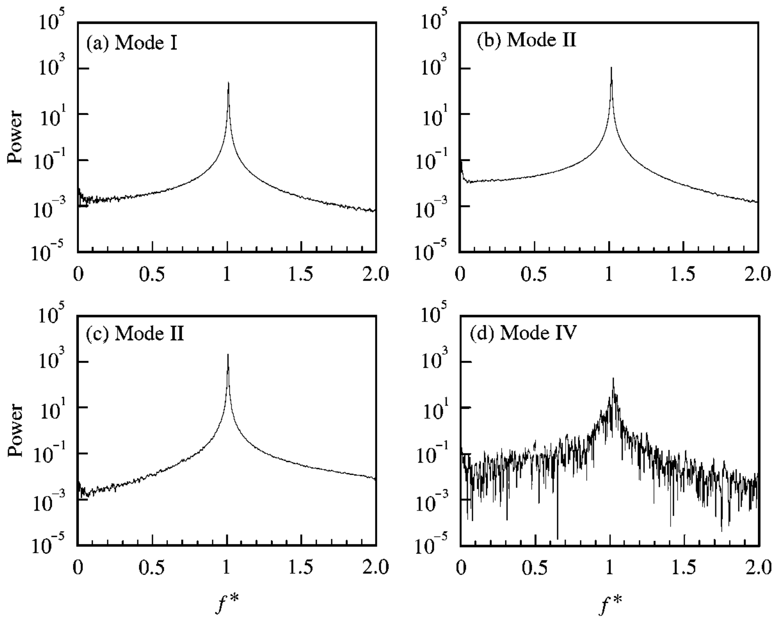

Figure 5. Position spectra in the different sphere oscillation modes for $m^* = 940$. Interestingly, although Mode IV has a low "periodicity", the vibration frequency (f) remains close to the natural frequency (f_N) i.e. $f^* = f/f_N \approx 1{\cdot}0$.

intermittent vibrations may be contrasted clearly with the periodic vibrations of Mode III, also shown in the figure, for $U^* = 30$.

A measure of the periodicity of the displacement may be given by plotting what we call the "periodicity" $= (\sqrt{2}\, y_{rms}/y_{max})$, as a function of velocity, U^*. A purely sinusoidal function has a value of "periodicity" equal to 1·0, and so the vibration Modes I, II and III are evidently strongly periodic, as also shown by the spectra in Figure 5. Mode IV, as expected has a low "periodicity", but interestingly, despite the bursts of large-amplitude vibration for this mode, the vibration frequency remains very close to the natural frequency throughout the range of velocity up to at least $U^* = 300$, as shown by the typical spectrum in Figure 5. Clearly, the much higher principal vortex shedding frequency (around 40–50 times the vibration frequency) is not itself responsible for these large vibrations! The origin of these large transient bursts of vibration remains unknown.

5. CONCLUDING REMARKS

In this paper, we present evidence for the existence of an unforeseen, highly periodic mode of vortex-induced vibration for a tethered sphere, which occurs at speeds far above what might be expected, based on classical "lock-in". The sphere appears to oscillate at large amplitudes, which can be up to one diameter for spheres of moderate mass ratio ($m^* \sim 10$), over a broad range of normalized velocities, $U^* = 20$–40. Vibration modes of a tethered sphere, which might be explained in terms of classical lock-in of the vortex frequency with the body frequency, have been discovered in Govardhan & Williamson (1997, 2001), and defined there as Modes I and II.

However, for the present high-speed "Mode III", the principal vortex shedding frequency is from 3 to 8 times the body oscillation frequency, and so the classical lock-in of frequencies cannot explain this vibration mode. Nevertheless, in order for these remarkably periodic vibrations to occur, there must be a component of fluid force that is exciting the body at its oscillation frequency. One may suggest that the body vibration is causing some modulation to the vortex formation sufficient to provide a fluid force at the vibration frequency, and with the right phase (between force and displacement) to excite such vibration. Indications of this are suggested by further work using force, displacement and vorticity measurements in Govardhan & Williamson (2001). One might suggest that the high-speed Mode III vibrations are the result of a "movement-induced vibration" of the type classified in Naudasher & Rockwell (1993), such as flutter and galloping, where the body dynamics may be explained in terms of quasi-steady analysis. In our case, the body is spherically symmetric, so a direct link is not evident. A further unsteady mode of vibration (defined here as Mode IV), characterized by intermittent burst of large amplitude, is found for extremely high velocities beyond $U^* = 100$, and the origin of this mode remains unknown.

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